Hamilton completion and the path cover number of G(n,p)

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The path cover number of a graph G is the minimum number of vertex disjoint paths required to cover its vertex set. If G is not Hamiltonian, it is also equal to the minimum number of edges whose addition to G results on a Hamiltonian graph. Since every path can cover at most two vertices with degree 1, clearly the number of paths in a path covering is at least half the number of degree 1 vertices. We show that when $G \sim G(n, c/n)$ and c is large enough, with high probability this trivial lower bound is essentially tight as a function of c.