The n-queens problem

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The n-queens problem asks how many ways there are to place n queens on an $n \times n$ chessboard so that no two queens can attack one another, and the toroidal n-queens problem asks the same question where the board is considered on the surface of a torus. Let $Q(n)$ denote the number of n-queens configurations on the classical board and $T(n)$ the number of toroidal n-queens configurations. The toroidal problem was first studied in 1918 by Pólya who showed that $T(n) > 0$ if and only if $n \equiv 1, 5 \mod 6$. Much more recently Luria showed that $T(n) \leq ((1 + o(1))ne^{-3})^n$ and conjectured equality when $n \equiv 1, 5 \mod 6$. We prove this conjecture, prior to which no non-trivial lower bounds were known to hold for all (sufficiently large) $n \equiv 1, 5 \mod 6$. We also show that $Q(n) \geq ((1+o(1))ne^{-3})^n$ for all $n \in \mathbb{N}$ which was independently proved by Luria and Simkin and, combined with our toroidal result, completely settles a conjecture of Rivin, Vardi and Zimmerman regarding both $Q(n)$ and $T(n)$.

In this talk we'll discuss some of the methods used to prove these results. A crucial element of this is translating the problem to one of counting matchings in a 4-partite 4-uniform hypergraph. Our strategy combines a random greedy algorithm to count 'almost' configurations with a complex absorbing strategy that uses ideas from the methods of randomised algebraic construction and iterative absorption.

This is joint work with Peter Keevash.