Schur properties of randomly perturbed sets

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A set A of integers is said to be Schur if any two-colouring of A results in monochromatic x, y and z with $x + y = z$. We study the following problem: how many random integers from $[n]$ need to be added to some $A \subseteq [n]$ to ensure with high probability that the resulting set is Schur? Hu showed in 1980 that when $|A| > 4n/5$, no random integers are needed, as A is already guaranteed to be Schur. Recently, Aigner-Horev and Person showed that for any dense set of integers $A \subseteq [n]$, adding $\omega(n^{1/3})$ random integers suffices, noting that this is optimal for sets A with $|A| \leq n/2$. We close the gap between these two results by determining how many random integers are needed to make a set $A \subseteq [n]$ Schur when $n/2 < |A| < 4n/5$, and initiate the study of perturbing sparse sets of integets A by providing nontrivial upper and lower bounds for the number of random integers needed in this case.

This is joint work with Charlotte Knierim and Patrick Morris.