

**An asymptotically optimal threshold bias for some $(a : b)$
Maker-Breaker games**

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In this paper, we study the $(a : b)$ Maker-Breaker minimum-degree- K game played on the complete graph (where $K = K(n)$). The aim of the Maker is to occupy the edges of a subgraph of a complete graph with minimum degree K , while the Breaker aims to prevent the Maker achieving his goal. We present a winning-strategy for the Maker leading to a bound that generalizes Gebauer and Szabó's bound in the $(1 : b)$ case. Moreover, we determine the asymptotic optimal generalized threshold bias $b_0(a)$ for this game to be $\frac{an}{a+\ln(n)}$ if $a = o\left(\sqrt{\frac{n}{\ln(n)}}\right)$ and $K = o(\ln(n))$, and n if $a = \Omega\left(\sqrt{\frac{n}{\ln(n)}}\right)$ and $K = o(a)$, by analyzing a Breaker strategy. Our bound for the Breaker is asymptotically the same as the Maker bound presented by Hefetz et al. (2012) for the $(a : b)$ connectivity game. This answers the open problem stated by these authors. We study also the $(a:b)$ Hamiltonicity game in which the Maker's goal is to create a Hamiltonian cycle. We determine the asymptotic optimal generalized threshold bias $b_0(a)$ for this game as well.