Expansion in the giant component of the percolated hypercube

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As in the case of the binomial random graph, it is known that the behaviour of a random subgraph of a d-dimensional hypercube, where we include each edge independently with probability p, undergoes a phase transition when p is around $1/d$. More precisely, answering a question of Erdős and Spencer, it was shown by Ajtai, Komlós and Szemerédi that significantly above this value of p, in the supercritical regime, whp this random subgraph has a unique 'giant' component, whose order is linear in the order of the hypercube.

In the binomial random graph much more is known about the complex structure of this giant component, much which can be deduced from more recent results about the likely expansion properties of the giant component. We show that whp the giant component L of a supercritical random subgraph of the d-dimensional hypercube has reasonably good expansion properties, and use this to deduce some structural information about L. In particular this leads to polynomial (in d) bounds on the diameter of L and the mixing time of a random walk on L, answering questions of Pete, and of Bollobás, Kohayakawa, and Łuczak.

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