Hypergraph removal with polynomial bounds

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Given a fixed k-uniform hypergraph F, the F-removal lemma states that every hypergraph with few copies of F can be made F-free by the removal of few edges. Unfortunately, for general F, the constants involved are given by incredibly fast growing Ackermann-type functions. It is thus natural to ask for which F can one prove removal lemmas with polynomial bounds. One trivial case where such bounds can be obtained is when F is k-partite. Alon proved that when $k = 2$ (i.e. when dealing with graphs), only bipartite graphs have a polynomial removal lemma. Kohayakawa, Nagle and Rodl conjectured in 2002 that Alon's result can be extended to all $k > 2$, namely, that the only k-graphs F for which the hypergraph removal lemma has polynomial bounds are the trivial cases when F is k-partite. We prove this conjecture. Joint work with Asaf Shapira.