On n-saturated closed graphs, or randomness in the service of her majesty logic

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One of the most important results in the theory of random graphs is given by Erdős and Rényi (1963) probabilistic construction of countable universal homogeneous graph, called from this reason *the random graph*. The random graph is obtained, with probability 1, as a limit of the $G(n,p)$ model where n tends to ∞, while $p \in (0,1)$ is fixed. On the other hand it is unique $\aleph_0$-saturated countable graph: any finite subgraph extends in each possible way.

Here we focus on topological graphs on the Cantor space 2^ω. Geschke proved that there is a clopen graph on 2^ω which is 3-saturated, but the clopen graphs on 2^ω do not even have infinite subgraphs that are 4-saturated. It is also known that there is no closed graph on 2^ω which is $\aleph_0$-saturated. We complete this picture by proving that for every $n \in \mathbb{N}$ there is an n-saturated closed graph 2^ω. The key lemma is based on a probabilistic argument. The final construction is an inverse limit of finite graphs.

This is joint work with Szymon Głąb (TUL, Łódź).