

Phase Transitions in Geometric Random Graphs on 2-dim Torus: Critical Phase

Vasilii Goriachkin and Tatyana Turova

Centre for Mathematical Sciences, Faculty of Science, Lund University, Solvegatan 18, 22100, Lund, Sweden.

Abstract

We consider random graphs on the set of N^2 vertices placed on the discrete 2-dimensional torus. The edges between pairs of vertices are independent, and their probabilities decay with the distance between the vertices. The probabilities of connections are scaled with the total number of vertices via distance in such a way that the graph has a finite expectation of degree when $N \rightarrow \infty$, and model becomes an example of inhomogeneous random graphs, notably not of rank 1. The reported results on the sub- and the super-critical phases show that this model exhibits phase transition in change of the size of the largest connected component strikingly similar to the one in the classical Erdős-Rényi random graph model.

While the sub- and super-critical cases of the model are well understood (and moreover in any dimension), we treat the critical phase for the first time. We derive a diffusion approximation for the scaled size of the largest connected component in the critical case, discovering again similarities with the classical random graph.

It can be noticed that the critical phase in the graphs beyond rank 1 is still practically not explored. It also remains a challenge to treat the considered model in higher dimensions.