Blowup Ramsey numbers

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Given graphs G and H, we say $G \xrightarrow{r} H$ if every r-colouring of the edges of G contains a monochromatic copy of H. Let $H[t]$ denote the t-blowup of H. The blowup Ramsey number $B(G \xrightarrow{r} H; t)$ is the minimum n such that $G[n] \xrightarrow{r} H[t]$. Fox, Luo and Wigderson refined an upper bound of Souza, showing that, given G, H and r such that $G \xrightarrow{r} H$, there exist constants $a = a(G, H, r)$ and $b = b(H, r)$ such that for all $t \in \mathbb{N}$, $B(G \xrightarrow{r} H; t) \leq ab^t$. They conjectured that there exist some graphs H for which the constant a depending on G is necessary. We prove this conjecture by showing that the statement is true in the case of H being 3-chromatically connected, which in particular includes triangles. On the other hand, perhaps surprisingly, we show that for forests F, the function $B(G \xrightarrow{r} F; t)$ is independent of G.
This is joint work with António Girão.