

Defective colouring of hypergraphs

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Erdős and Lovász originally proved the local lemma to show that every $(r + 1)$ -uniform hypergraph G with maximum degree Δ has chromatic number $O(\Delta^{1/r})$. We prove the following generalisation of this result. A *d-defective colouring* of a hypergraph is a vertex-colouring in which every vertex is in at most d monochromatic edges (so $d = 0$ is exactly a proper colouring). We prove that every $(r + 1)$ -uniform hypergraph G with maximum degree Δ has a d -defective colouring using at most

$$100 \left(\frac{\Delta}{d + 1} \right)^{1/r}$$

colours. This is tight up to the leading constant. Our proof uses a semirandom argument together with a sunflower decomposition trick.

This is joint work with António Girão (Oxford), Alex Scott (Oxford), and David Wood (Monash).