

The Turán density of three-uniform tight cycles

Nina Kamčev

The topic of this talk are Turán-type problems for three-uniform tight cycles $C_\ell^{(3)}$ with $3 \nmid \ell$. The Turán density of an r -graph F , denoted $\pi(F)$, is the limit of the maximum density of an n -vertex r -graph not containing F , as $n \rightarrow \infty$. There are very few examples of three-uniform hypergraphs with known Turán density.

Mubayi and Rödl (2002) found an iterated construction of a $C_5^{(3)}$ -free hypergraph with edge density asymptotically $2\sqrt{3}-3 \approx 0.464$, which is conjectured to be optimal for $C_5^{(3)}$. We prove that any sufficiently long non-three-partite tight cycle has the above conjectured Turán density. More precisely, if ℓ is sufficiently large and not divisible by 3, then $\pi(C_\ell) = 2\sqrt{3} - 3$. One of our main tools, which may be of independent interest, is a three-uniform analogue of the statement ‘a graph is bipartite if and only if it does not contain an odd cycle’.

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