The (p:q)-Game on Hypergraphs

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We introduce the (p:q)-Game played on a hypergraph $H = (V, E)$ with $|E| = m$ by two players, Balancer and Unbalancer. In each round of the game Balancer selects p elements of the vertex set V before Unbalancer selects q elements of the vertex set V. The game is played until all vertices are selected or one player has achieved his win condition. Balancer's aim is to have around $\frac{p}{p+q}|e_j|$ vertices in every edge $e_j \in E$. Unbalancer tries to prevent Balancer from achieving this goal. Balancer's winning condition can be formulated in the following way. For every edge $e_j \in E$ let $b_j \geq 0$ be the allowed deviation. Balancer wins, if his selected number of vertices t_j in edge e_j satiesfies $\left|t_j - \frac{p}{p+q}|e_j|\right| \geq b_j$ for all edges e_j. We are looking for the smallest possible deviation b_j. Alon et al. (2005) proved that for the (1:1)-Game $b_j \geq \sqrt{2\ln(2m)|e_j|}$. They posed the analysis of the general (p:q)-Game as a challenging open problem. In fact, no nontrivial deviation was known. We give a polynomial time algorithm for Balancer to win the game provided the allowed deviation is large enough. In particular, Balancer wins the general (p:q)-Game for $b_j \geq \sqrt{2\ln(2^{\max(p,q)+1}|e_j|m)|e_j|}$.

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