**Distribution of Missing Sums in Sumsets**

*Dylan King*

Given a finite set of integers $A$, its sumset is $A + A := \{a_i + a_j \mid a_i, a_j \in A\}$. We examine $|A + A|$ as a random variable, where $A \subset I_n = [0, n-1]$, the set of integers from 0 to $n - 1$, so that each element of $I_n$ is in $A$ with a fixed probability $p \in (0, 1)$. In this case $|A + A|$ is a random variable with $0 \le |A + A| \le 2n - 1$ and its distribution can be studied in detail. Martin and O'Bryant studied the behavior of $|A + A|$ when $p = 1/2$ and found a closed form for $\mathbb{E}[|A+A|]$. Lazarev, Miller, and O'Bryant extended the result to find a numerical estimate for $\mathrm{Var}(|A + A|)$ and bounds on the number of missing sums in $A + A$, $m_{n\,;p}(k) := \mathbb{P}(2n - 1 - |A + A| = k)$. Their primary tool was a graph-theoretic framework which we now generalize to provide a closed form for $\mathbb{E}[|A + A|]$ and $\mathrm{Var}(|A + A|)$ for all $p \in (0, 1)$ and establish good bounds for $\mathbb{E}[|A + A|]$ and $m_{n\,;p}(k)$.
We continue to investigate $m_{n\,;p}(k)$ by studying $m_p(k) = \lim_{n\to\infty} m_{n\,;p}(k)$, proven to exist by Zhao. Lazarev, Miller, and O'Bryant proved that, for $p = 1/2$, $m_{1/2}(6) > m_{1/2}(7) < m_{1/2}(8)$. This distribution is not unimodal, and is said to have a "divot" at 7. We report results investigating this divot as $p$ varies, and through a combination of theoretical and numerical analysis, prove that for $p \ge 0.68$ there is a divot at 1; that is, $m_p(0) > m_p(1) < m_p(2)$.