Minimum degree of asymmetric Ramsey-minimal graphs.

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A graph G is q-Ramsey for a q-tuple of graphs $(H_1, \ldots, H_q)$, denoted by $G \to_q (H_1, \ldots, H_q)$, if every q-colouring $c : E(G) \to [q]$ contains a monochromatic copy of H_i in colour i, for some $i \in [q]$. The graph G is called q-Ramsey-minimal for $(H_1, \ldots, H_q)$ if it is q-Ramsey for $(H_1, \ldots, H_q)$ but no proper subgraph of G possesses this property. Let $s_q(H_1, \ldots, H_q)$ denote the smallest minimum degree of G over all graphs G that are q-Ramsey-minimal for $(H_1, \ldots, H_q)$.

The study of the parameter s_2 was initiated by Burr, Erdős and Lovász in 1976 when they showed that for cliques, $s_2(K_k, K_t) = (k-1)(t-1)$. In the past two decades the parameter s_q has been studied extensively, focusing on its symmetric version with $H_i = H$ for all i (H being a clique, a cycle, certain bipartite graph or from some sporadic classes of graphs). We present three new results in the asymmetric setting, two exact results with 2 colours for the parameters $s_2(K_k, C_\ell)$ and $s_2(C_k, C_\ell)$ (where C_ℓ is a cycle of length ℓ), and find various upper bounds on $s_q(K_k, \ldots, K_k, C_\ell, \ldots, C_\ell)$, depending on the range of parameters.

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