On Symmetric Intersecting Families

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A family F of vectors in $[k]^n = \{1, 2, \ldots, k\}^n$ is called *intersecting* if any two vectors in F are equal in at least one coordinate. It is clear that the maximal size of an intersecting family $F \subset [k]^n$ is k^{n-1}, attained by the family $F = \{x : x_1 = 1\}$. In 2019, Eberhard, Kahn, Narayanan and Spirkl showed that if the family is also required to be symmetric (i.e., invariant under a transitive group of permutations of the coordinates), then the maximal size M of the family drops to $o(k^n)$. Eberhard et al. presented quantitative lower and upper bounds on this maximal size: $k^{n-\sqrt{n}} < M < k^n/n^{c/k}$, and conjectured that the "correct" value is k^{n-n^δ} for some $\delta > 0$.

Using tools of analysis of Boolean functions, we present improved lower and upper bounds for different ranges of n, k. In particular, we show that $M > k^n/n^{k \log k}$, thus refuting the conjecture of Eberhard et al. In addition, we show that when $k > C \log n$ (for a sufficiently large constant C), $M < k^n/n^c$, and that when $k > C \log^2 n$, we have $M < k^n/n^{c \log k}$.