Graphs with large minimum degree and no small odd cycles are three-colourable

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This talk is based on joint work with Julia Böttcher, Nora Frankl, Olaf Parczyk and Jozef Skokan.

Let $\mathcal{F}$ be a fixed family of graphs. The *homomorphism threshold* of $\mathcal{F}$ is the infimum of those α for which there exists an $\mathcal{F}$-free graph $H(\mathcal{F},\alpha)$, such that every $\mathcal{F}$-free graph on n vertices of minimum degree αn is homomorphic to $H(\mathcal{F},\alpha)$. Letzter and Snyder showed that the homomorphism threshold of $\{C_3, C_5\}$ is $1/5$. They found explicit graphs $H(\mathcal{F},\alpha)$ for $\alpha \geq \frac{1}{5}+\varepsilon$, which were in addition 3-colourable. Thus, their result also implies that $\{C_3, C_5\}$-free graphs of minimum degree at least $(\frac{1}{5}+\varepsilon)n$ are 3-colourable. For longer cycles, Ebsen and Schacht showed that the homomorphism threshold of $\{C_3, C_5, \ldots, C_{2\ell-1}\}$ is $\frac{1}{2\ell-1}$. However, their proof does not imply a good bound on the chromatic number of $\{C_3, \ldots, C_{2\ell-1}\}$-free graphs of minimum degree $(\frac{1}{2\ell-1}+\varepsilon)n$. Answering a question of Letzter and Snyder, we prove that such graphs are 3-colourable.