Oriented discrepancy of Hamilton cycles

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One of the sufficient conditions for Hamiltonicity in graphs, obtained by Dirac in 1952, asserts that every n-vertex graph of minimum degree at least n/2 ("Dirac graph") is Hamiltonian. We conjecture the following generalization: in any orientation of the edges of an n-vertex graph with minimum degree $\delta \geq n/2$ there exists a Hamilton cycle in which at least delta edges are pointing forward. In this talk, I will present a proof for an approximate version of this conjecture, according to which a minimum degree of (n+O(k))/2 suffices to guarantee a Hamilton cycle with at least (n+k)/2 edges pointing forward. I will also discuss an analogous problem for random graphs, showing that above the Hamiltonicity threshold, any orientation contains, with high probability, an "almost-directed" Hamilton cycle, namely, one in which almost all of the edges point in the same direction.

The talk is based on joint work with Lior Gishboliner and Michael Krivelevich.