Multicolor Turán numbers

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Abstract

We consider a natural generalisation of Turán's forbidden subgraph problem and the Ruzsa-Szemerédi $(6,3)$ problem. Let $\mathrm{ex}_F(n,G)$ denote the maximum number of edge-disjoint copies of a fixed graph F that can be placed on an n-vertex ground set without forming a subgraph G whose edges are from different F-copies. One can also think of this as placing graphs F of distinct colors and want to avoid a multicolor G.
We characterize those pairs (F,G) for which the order of magnitude of $\mathrm{ex}_F(n,G)$ is quadratic and prove several asymptotic results using various tools from the Szemerédi regularity lemma and supersaturation to graph packing results and additive number theory. Finally we show some applications.
The main results presented will be:

Theorem 0.1 (Imolay, Karl, Nagy and Váli [1]). $\mathrm{ex}_F(n,G)=\Theta(n^2)$ *if and only if there is no homomorphism from* G *to* F*, and* $\mathrm{ex}_F(n,G)=o(n^2)$ *otherwise.*

Theorem 0.2 (Imolay, Karl, Nagy and Váli [1]). *If* $\chi(F)<\chi(G)$*, then*

$$\mathrm{ex}_F(n,G)\sim\frac{1-\frac{1}{\chi(G)-1}}{2e(F)}n^2.$$

Theorem 0.3 (Kovács, Nagy [2]). *For every* $t\geq r\geq 3$*, we have*

$$n^2e^{-O(\sqrt{\log n})}\leq \mathrm{ex}_{K_t}(n,K_r)=o(n^2)$$

and in general, if F *and* G *are connected graphs of girth* 3*, then* $n^2e^{-O(\sqrt{\log n})}\leq \mathrm{ex}_F(n,G)$.

This is joint work with A. Imolay, J. Karl, B. Váli and B. Kovács.

References

[1] Imolay, A., Karl, J., Nagy, Z. L. and Váli, B. (2021). Multicolour Turán numbers. Discrete Mathematics, 345(9), 112976.

[2] Kovács, B., Nagy, Z.L. (2022) Multicolor Turán numbers II. - a generalization of the Ruzsa-Szemerédi theorem and new results on cliques and odd cycles.