FUNCTIONAL CENTRAL LIMIT THEOREMS FOR LOCAL STATISTICS OF SPATIAL BIRTH-DEATH PROCESSES IN THE THERMODYNAMIC REGIME

EFE ONARAN[1,*]

[1] *Viterbi Faculty of Electrical and Computer Engineering, Technion–Israel Institute of Technology*
[*] *efeonaran@campus.technion.ac.il*

Spatial birth-death processes are continuous time Markov chains with state spaces of counting measures in a metric space, typically $\mathbb{R}^d$. Random geometric graphs, on the other hand, are static models generated by points randomly scattered in Euclidian space, for which pairs of points in close proximity are assumed to create edges. The statistical properties of local, non-negative functionals defined on random geometric graphs, such as numbers of edges or subgraphs isomorphic to a given graph, along with the techniques to derive them, are generally well understood.

In this talk, we will present *functional* central limit theorems for a broad class of local, non-negative functionals defined on subsets of points in spatial birth-death processes. The functionals are parametrized by the size of their *local support*, and our focus in this talk will be on the regime where the expected number of particles in shrinking supports converges to a constant as the overall density of particles goes to infinity. This is generally called the *thermodynamic regime* in the random geometric graph literature. We prove that subject to mild assumptions, normalized sums of these spatial functionals (summed over all subsets of the underlying particles alive at a given time) converge weakly to weighted superpositions of independent Ornstein-Uhlenbeck processes. In particular, this superposition is infinite when the functionals satisfy a certain neighbourhood vacancy condition, an example of which is the isolated subgraph (*component*) count. Further examples of practical interest will be discussed.

As for our methods, we rely heavily on the modern theory of normal approximation for stabilizing functionals. Historically, central limit theorems in stochastic geometry have been based either on normal approximation theory for martingale differences or on the classical Stein method. Recent developments for Poisson space functionals, fusing the Malliavin calculus and the Stein method led to normal approximation theorems easily applicable to point processes in more general spaces, while also providing improved convergence rates. In this work, we extend the applications of this new theory by modelling the temporally evolving spatial birth-death process as a static *marked* point process. We employ the marked point process representation also to calculate the limiting covariance and bounds on the higher moments needed to establish the limit theorems at the process level.

To be more concrete, our proofs of weak convergence is defined with respect to the Skorokhod topology defined in the space of càdlàg (right continuous with left limits) functions on $[0, \infty)$, which is the time dimension. As per usual in weak convergence results for probability measures in metric spaces, our proofs are composed of two main steps. First, we establish the convergence of finite dimensional distributions using the recent Malliavin-Stein bounds for the normal convergence and the marked representation. Following this, we prove *tightness* of the sequence of probability measures regarding the functionals of interest to us. For this we use a classical result establishing tightness for processes with bounded moments, for which we again use the marked process representation we introduced. We will in our talk go through the main ideas as well as the necessary background for the tools used in the proofs.

This is a joint work with Omer Bobrowski (Technion and Queen Mary University of London) and Robert J. Adler (Technion).