1-independent percolation in $\mathbb{Z}^2 \times K_n$

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(This talk is based on joint work with Victor Falgas-Ravry.)

A random graph model on a host graph H is said to be 1-*independent* if for every pair of vertex-disjoint subsets A, B of $E(H)$, the state of edges (absent or present) in A is independent of the state of edges in B. For an infinite connected graph H, the 1-*independent critical percolation probability* $p_{1,c}(H)$ is the infimum of the $p \in [0,1]$ such that every 1-independent random graph model on H in which each edge is present with probability at least p almost surely contains an infinite connected component.

Balister and Bollobás observed in 2012 that $p_{1,c}(\mathbb{Z}^d)$ is nonincreasing and tends to a limit in $[\frac{1}{2}, 1]$ as $d \to \infty$. They asked for the value of this limit. We make progress towards this question by showing that

$$\lim_{n\to\infty} p_{1,c}(\mathbb{Z}^2 \times K_n) = 4 - 2\sqrt{3} = 0.5358\ldots\ .$$

In fact, we show that the equality above remains true if the sequence of complete graphs K_n is replaced by a sequence of weakly pseudorandom graphs on n vertices with average degree $\omega(\log n)$. We conjecture that the equality also remains true if K_n is replaced instead by the n-dimensional hypercube Q_n. This latter conjecture would imply the answer to Balister and Bollobás's question is $4 - 2\sqrt{3}$.