On a generalisation of Dirac's Theorem to uniformly dense hypergraphs

Simón Piga

We study tight Hamilton cycles in quasirandom hypergraphs with minimum degree at least $\Omega(n^{k-1})$. For 3-uniform hypergraphs and different notions of quasirandomness these type of problems were studied previously by Aigner-Horev and Levy, Gan and Han, and the authors. We generalise those results for k-uniform hypergraphs.

For one notion of quasirandomness and under a minimum degree condition of $\Omega(n^{k-1})$, we obtain an asymptotically optimal density threshold that enforces the existence of a tight Hamilton cycle. Moreover, we prove that under the same minimum degree conditions, for stronger notions of quasirandomness, any arbitrarily small density is already enough to ensure the existence of such a cycle. Additionally, for weaker notions, we provide examples of kuniform hypergraphs with quasirandom density almost 1 and subject to the same minimum degree condition, that do not contain tight Hamilton cycles.