The 1–2–3 Conjecture – recent progresses

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Abstract

A simple graph more often than not contains adjacent vertices with equal degrees. This in particular holds for all pairs of neighbours in regular graphs, while a lot such pairs can be expected e.g. in many random models. Is there a universal constant K, say $K = 3$, such that one may always dispose of such pairs from any given connected graph with at least three vertices by blowing its selected edges into at most K parallel edges? This question was first posed in 2004 by Karoński, Łuczak and Thomason, who equivalently asked if one may assign weights $1, 2, 3$ to the edges of every such graph so that adjacent vertices receive distinct weighted degrees – the sums of their incident weights. This basic problem is commonly referred to as the 1–2–3 Conjecture nowadays, and has been addressed in multiple papers, constituting wealthy source of inspiration for developing new techniques and intriguing further questions.

The best result concerning the 1–2–3 Conjecture asserts that weights $1, 2, 3, 4, 5$ are always sufficient. Within the talk we shall discuss recent progresses concerning this concept. In particular, the mentioned general result can be improved for regular graphs [2], while the conjecture itself has been confirmed for graphs with minimum degree δ large enough, i.e. for $\delta = \Omega(\log \Delta)$, where Δ denotes the maximum degree of the graph [3]. We shall also mention recent impressive progress concerning the list generalization of this concept from [1,4], based on applications of Alon's Combinatorial Nullstellensatz.

References:

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[4] X. Zhu, Every nice graph is $(1, 5)$-choosable, *arXiv:2104.05410*.