COUNTING SPANNING TREES IN DIRAC GRAPHS

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Let T be a oriented tree on n vertices with maximum degree at most $e^{\gamma\sqrt{\log n}}$ for some $\gamma > 0$. If G is a digraph on n vertices with minimum semidegree $\delta^0(G) \geq (\frac{1}{2} + \varepsilon)n$ for some $\varepsilon > 0$, then G contains T as a spanning tree, as recently shown by Kathapurkar and Montgomery. This generalizes the corresponding result by Komlós, Sárközy and Szemerédi for graphs. We investigate the natural question how many copies of T the digraph G contains. We prove that every such G contains at least $\frac{1}{2^{n-1}} \frac{n!}{|\mathrm{Aut}(T)|}(1 - o(1))^n$ copies of T, which is optimal. This implies the analogous result in the undirected case.