The number of n-queens configurations

Michael Simkin

The n-queens problem is to determine $Q(n)$, the number of ways to place n mutually non-threatening queens on an $n \times n$ board. We show that there exists a constant $1.94 < a < 1.9449$ such that $Q(n) = ((1 + o(1))ne^{(-a)})^n$. The constant a is characterized as the solution to a convex optimization problem in $P([-1/2, 1/2]^2)$, the space of Borel probability measures on the square. The chief innovation is the introduction of limit objects for n-queens configurations, which we call "queenons". These are a convex set in $P([-1/2, 1/2]^2)$. We define an entropy function that counts the number of n-queens configurations approximating a given queenon. The upper bound uses the entropy method of Radhakrishnan and Linial–Luria. For the lower bound we describe a randomized algorithm that constructs a configuration near a prespecified queenon and whose entropy matches that found in the upper bound. The enumeration of n-queens configurations is then obtained by maximizing the (concave) entropy function over the space of queenons.