

Cohomology groups of non-uniform random simplicial complexes

Philipp Sprüssel

Starting from the classical result that $p = \frac{\ln n}{n}$ is a sharp threshold for connectedness of the binomial random graph $G(n, p)$, there have been multiple approaches to study higher-dimensional analogues. In particular, random hypergraphs and random simplicial complexes have received considerable attention.

We study a model of random simplicial complexes that is based on a non-uniform random hypergraph: Independently for each set S of vertices, let S form an edge with a probability that depends on the size of S . This hypergraph then gives rise to a simplicial complex by taking the downward-closure. For a fixed positive integer j , j -connectedness of the random simplicial complex can now be defined via the vanishing of cohomology groups up to dimension j .

We prove a sharp threshold for j -connectedness of this model of random simplicial complexes. In addition, we prove a hitting time result that relates j -connectedness to the vanishing of the last minimal obstruction, and we determine the asymptotic behaviour of cohomology groups inside the critical window around the time of the phase transition.

This talk is based on joint work with N. Del Giudice, O. Cooley, and M. Kang.