Ramsey goodness of bounded degree trees versus general graphs

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Given two graphs G and H, the *Ramsey number* $R(G, H)$ is defined as the smallest integer N so that any red/blue colouring of the edges of the complete graph K_N contains either a red copy of G or a blue copy of H. In 1981, Burr proved a general lower bound which states that for any connected graph G and any graph H such that $|G| \geq \sigma(H)$, we have $R(G, H) \geq (|G| - 1)(\chi(H) - 1) + \sigma(H)$, where $\sigma(H)$ is the smallest possible size of a colour class in any $\chi(H)$-colouring of H. Motivated by this result, a graph G is called H-good if $R(G, H) = (|G| - 1)(\chi(H) - 1) + \sigma(H)$. Since Burr and Erdős introduced this notion of Ramsey goodness in 1983, it has received considerable interest and has been extensively studied.

A recent result of Balla, Pokrovskiy and Sudakov showed that for any fixed Δ, k, there exists a constant $C_{\Delta,k}$ such that for any graph H with $\chi(H) = k$, every tree T with maximum degree at most Δ and $|T| \geq C_{\Delta,k}|H| \log^4 |H|$ is H-good. Moreover, they showed that the dependency between $|T|$ and $|H|$ is tight up to the log factor and conjectured that the log factor can be removed. In this joint work with Richard Montgomery and Matías Pavez-Signé, we confirm this conjecture by proving the same statement but with $|T| \geq C_{\Delta,k}|H|$ instead.